Grade 5: Unit 3
More Operations on Fractions
Course Philosophy/Description

In mathematics, students will learn to address a range of tasks focusing on the application of concepts, skills and understandings. Students will be asked to solve problems involving the key knowledge and skills for their grade level as identified by the NJSL; express mathematical reasoning and construct a mathematical argument and apply concepts to solve model real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

Fifth grade Mathematics consists of the following domains: Operations and Algebraic Thinking (OA), Number and Operations in Base Ten (NBT), Number and Operations-Fractions (NF), Measurement and Data (MD), and Geometry (G). In fifth grade, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards (NJSLS). The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
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<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
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<tbody>
<tr>
<td>1</td>
<td>Multiply fractions by whole numbers and fractions by fractions, drawing visual models such as area models with the correct use of tiling, to represent products showing ((a/b) \times (c/d) = ab(1/bd)), and creating story contexts.</td>
<td>5.NF.B.4a,b</td>
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<tr>
<td>2</td>
<td>Explain how a product is related to the magnitude of the factors, including cases in which one factor is a fraction greater than 1 and cases in which one factor is a fraction less than 1.</td>
<td>5.NF.B.5</td>
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<tr>
<td>3</td>
<td>Solve real-world problems involving multiplication of fractions (including mixed numbers), using visual fraction models or equations to represent the problem.</td>
<td>5.NF.B.6</td>
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<tr>
<td>4</td>
<td>Divide a unit fraction by a non-zero whole number and interpret by creating a story context or visual fraction model.</td>
<td>5.NF.B.7a*</td>
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<tr>
<td>5</td>
<td>Divide a whole number by a unit fraction and interpret by creating a story context or visual fraction model.</td>
<td>5.NF.B.7b*</td>
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<tr>
<td>6</td>
<td>Solve real-world problems involving division of unit fractions by whole numbers or whole numbers by unit fractions.</td>
<td>5.NF.B.7c*</td>
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<td>7</td>
<td>Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10; represent powers of 10 using whole-number exponents.</td>
<td>5.NBT.A.2*</td>
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<tr>
<td>8</td>
<td>Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; explain the reasoning used, relating the strategy to the written method.</td>
<td>5.NBT.B.7*</td>
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<tr>
<td>9</td>
<td>Convert standard measurement units within the same system (e.g., centimeters to meters) in order to solve multi-step problems.</td>
<td>5.MD.A.1</td>
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</tbody>
</table>

Instruction: 8 weeks
Assessment: 1 week
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)
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<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<td><strong>Collaborative Problem Solving</strong></td>
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<td>Connect Previous Knowledge to New Learning</td>
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<td>Making Thinking Visible</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<tr>
<td>Use of Multiple Representations</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Quick Writes</td>
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<tr>
<td>Pair/Trio Sharing</td>
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<td>Turn and Talk</td>
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<td>Charting</td>
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<td>Gallery Walks</td>
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<td>Small Group and Whole Class Discussions</td>
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<td>Student Modeling</td>
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<tr>
<td><strong>Analyze Student Work</strong></td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Interviews</td>
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<tr>
<td>Role Playing</td>
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<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Multiple Response Strategies</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Revoicing</td>
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<td>Marking</td>
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<td>Recapping</td>
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<td>Challenging</td>
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<tr>
<td>Pressing for Accuracy and Reasoning</td>
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<tr>
<td>Maintain the Cognitive Demand</td>
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## Educational Technology Standards

### 8.1.5.A.1, 8.2.5.D.3, 8.2.5.C.4

- **Technology Operations and Concepts**
  - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

  **Example:** Shepard Software, and Moby Max are websites that allow students to solve problems that promote critical and computational thinking.  [http://www.mobymax.com](http://www.mobymax.com)  [http://www.sheppardsoftware.com/math.htm](http://www.sheppardsoftware.com/math.htm)

- **Abilities for a Technological World**
  - Follow step by step directions to assemble a product or solve a problem.

  **Example:** Students can utilize digital resources to help them understand how to solve problems that require various steps, such as multiplying a fraction and a whole number.  [https://www.khanacademy.org/math/arithmetic-home/arithmetic-review-fractions/multiplying-fractions-and-whole-numbers](https://www.khanacademy.org/math/arithmetic-home/arithmetic-review-fractions/multiplying-fractions-and-whole-numbers)

- **Design**
  - Collaborate and brainstorm with peers to solve a problem evaluating all solutions to provide the best results with supporting sketches or models.

  **Example:** Games called “Equivalent Fractions” and “Concentration” allow you to create equivalent fractions by dividing and shading squares or circles and matching each fraction to its location on a number line.  [https://illuminations.nctm.org/Search.aspx?view=search&kw=fractions&gr=3-5](https://illuminations.nctm.org/Search.aspx?view=search&kw=fractions&gr=3-5)
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of specific tools and use tools to explore and deepen understanding of operations with fractions. Number lines will be used as tools to show students fractional representations between two whole numbers.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will communicate precisely using clear definitions and provide carefully formulated explanations when constructing arguments. Students will communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions. Students will ask probing questions to clarify or improve arguments. In this unit, comparison symbols such as <, >, = and fraction models are utilized to compare fractional values and defend arguments regarding their sizes.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or others.
Career Ready Practices

**Example:** Students will understand the meaning of a problem and look for entry points to its solution. They will analyze information, make conjectures, and plan a solution pathway. Students will monitor and evaluate progress and change course as necessary. In this unit, students will draw figures and identify the side lengths to find the perimeter and make adjustments based on reasonableness of the value represented and the context of the word problem.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

**Example:** Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify or improve arguments. In this unit, students will justify why certain shapes do not fit into specific subcategories based on their attributes such as squares and rectangles being quadrilaterals.
## WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Proficiency</th>
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</table>
| **6- Reaching** | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners

- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

### BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

<table>
<thead>
<tr>
<th>CONTENT INTEGRATION</th>
<th>KNOWLEDGE CONSTRUCTION</th>
<th>PREJUDICE REDUCTION</th>
<th>EQUITABLE PEDAGOGY</th>
<th>EMPOWERING SCHOOL CULTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use examples and content from a variety of cultures &amp; groups.</td>
<td>Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives &amp; biases.</td>
<td>Teachers implement lessons and activities to assert positive images of ethnic groups &amp; improve intergroup relations.</td>
<td>Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.</td>
<td>Using the other four dimensions to create a safe and healthy educational environment for all.</td>
</tr>
<tr>
<td>This unit / lesson is connected to other topics explored with students.</td>
<td>This unit / lesson provides context to the history of privilege and oppression.</td>
<td>This unit / lesson helps students question and unpack biases &amp; stereotypes.</td>
<td>The instruction has been modified to meet the needs of each student.</td>
<td>There are opportunities for students to connect with the community.</td>
</tr>
<tr>
<td>There are multiple viewpoints reflected in the content of this unit / lesson.</td>
<td>This unit / lesson addresses power relationships.</td>
<td>This unit / lesson helps students examine, research and question information and sources.</td>
<td>Students feel respected and their cultural identities are valued.</td>
<td>My classroom is welcoming and supportive for all students?</td>
</tr>
<tr>
<td>The materials and resources are reflective of the diverse identities and experiences of students.</td>
<td>This unit / lesson helps students to develop research and critical thinking skills.</td>
<td>The curriculum encourages discussion and understanding about the groups of people being represented.</td>
<td>Additional supports have been provided for students to become successful and independent learners.</td>
<td>I am aware of and sensitive to the needs of my students and their families.</td>
</tr>
<tr>
<td>The content affirms students, as well as exposes them to experiences other than their own.</td>
<td>This curriculum creates windows and mirrors* for students.</td>
<td>This unit / lesson challenges dominant perspectives.</td>
<td>Opportunities are provided for students to reflect on their learning and provide feedback.</td>
<td>There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.</td>
</tr>
</tbody>
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Culturally Relevant Pedagogy Examples

- **Use Problem-Based Learning Scenarios:** Present relatable real-world problems for your students to solve, explicitly referencing cultures and communities when applicable.

  **Example:** The teacher will begin the lesson by discussing with the class the differences in speed limits on highways, county roads, residential streets, or school streets. The class will record the various examples of speed limits on a chart using miles per hour and discuss the length of a mile. The teacher will explain that a foot is a unit of length that is used in U.S. and Britain. The teacher will also explain the conversion of feet to yards and mile to feet. The teacher will lead a discussion regarding the speed limit in Canada, explaining that in other countries people use a metric system of measurement. Explain the equivalence of one meter to kilometer and centimeters. The teacher will divide the class into partnerships. Some students will use a meter stick and others will use the yardstick. Students will measure different objects in the classroom. The last object to be measured can be an option for the student. Students that complete their measurements will convert the measurements on their worksheets and then discuss the difference between the two systems.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.

  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Use Learning Stations:** Provide a range of material by setting up learning stations.

  **Example:** Reinforce understanding of concepts and skills by promoting the learning through student interests and modalities, experiences and/or prior knowledge. Encourage the students to make choices in content based upon their strengths, needs, values and experiences. Providing students with choice boards will give them a sense of ownership to their learning and understanding.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.

  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures practice and cognates. Model to students that some vocabulary has multiple meanings. Have students create the Word Wall with their definitions and examples to foster ownership. Work with students to create a variety of sorting and match games of vocabulary words in this unit. Students can work in teams or individually to play these games for approximately 10-15 minutes each week. This will give students a different way of becoming familiar with the vocabulary rather than just looking up the words or writing the definition down.
SOCIAL AND EMOTIONAL LEARNING (SEL) COMPETENCIES

SELF-AWARENESS
The ability to accurately recognize one’s own emotions, thoughts, and values and how they influence behavior. The ability to accurately assess one’s strengths and limitations, with a well-grounded sense of confidence, optimism, and a “growth mindset.”
- Identifying Emotions
- Accurate Self-perception
- Recognizing Strengths
- Self-confidence
- Self-efficacy

SOCIAL AWARENESS
The ability to take the perspective of and empathize with others, including those from diverse backgrounds and cultures. The ability to understand social and ethical norms for behavior and to recognize family, school, and community resources and supports.
- Perspective-taking
- Empathy
- Appreciating Diversity
- Respect for Others

RESponsible Decision-Making
The ability to make constructive choices about personal behavior and social interactions based on ethical standards, safety concerns, and social norms. The realistic evaluation of consequences of various actions, and a consideration of the well-being of oneself and others.
- Identifying Problems
- Analyzing Situations
- Solving Problems
- Evaluating
- Reflecting
- Ethical Responsibility

SELF-MANAGEMENT
The ability to successfully regulate one’s emotions, thoughts, and behaviors in different situations — effectively managing stress, controlling impulses, and motivating oneself. The ability to set and work toward personal and academic goals.
- Impulse Control
- Stress Management
- Self-discipline
- Self-motivation
- Goal Setting
- Organizational Skills

RELATIONSHIP SKILLS
The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. The ability to communicate clearly, listen well, cooperate with others, resist inappropriate social pressure, negotiate conflict constructively, and seek and offer help when needed.
- Communication
- Social Engagement
- Relationship Building
- Teamwork
<table>
<thead>
<tr>
<th>SEL Competency</th>
<th>Examples</th>
<th>Content Specific Activity &amp; Approach to SEL</th>
</tr>
</thead>
</table>
| ✓ Self-Awareness  
Self-Management  
Social-Awareness  
Relationship Skills  
Responsible Decision-Making | Example practices that address Self-Awareness:  
• Clearly state classroom rules  
• Provide students with specific feedback regarding academics and behavior  
• Offer different ways to demonstrate understanding  
• Create opportunities for students to self-advocate  
• Check for student understanding / feelings about performance  
• Check for emotional wellbeing  
• Facilitate understanding of student strengths and challenges | Students begin the unit by drawing themselves and identifying the attributes they have that makes them a good mathematician. Students are encouraged to add more affirmative language as they learn additional math skills.  
Routinely tell students authentic reasons why you feel happy/optimistic for them and their future, including your optimism about their ability to succeed in a career in math, science, or technology. Encourage conversations about how they feel learning math will contribute to their success in the future. |
| ✓ Self-Management  
Self-Awareness  
Social-Awareness  
Relationship Skills  
Responsible Decision-Making | Example practices that address Self-Management:  
• Encourage students to take pride/ownership in work and behavior  
• Encourage students to reflect and adapt to classroom situations  
• Assist students with being ready in the classroom  
• Assist students with managing their own emotional states | Students reflect on barriers they may encounter when completing an assignment by asking clarifying questions, trying out others’ strategies and describing the approaches used by others when multiplying and dividing fractions.  
Students use their skills for focusing attention, managing stress and anxiety, and accomplishing goals in order to effectively participate in learning centers. |
| ✓ Social-Awareness  
Self-Management  
Self-Awareness  
Social-Awareness  
Relationship Skills  
Responsible Decision-Making | Example practices that address Social-Awareness:  
• Encourage students to reflect on the perspective of others | Teach lessons that develop mathematical discourse and engage students in purposeful sharing of mathematical ideas, reasoning and approaches using varied representations. |
<table>
<thead>
<tr>
<th>Self-Awareness</th>
<th>Example practices that address Relationship Skills:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Management</td>
<td>• Engage families and community members</td>
</tr>
<tr>
<td>Social-Awareness</td>
<td>• Model effective questioning and responding to students</td>
</tr>
<tr>
<td>✓ <strong>Relationship Skills</strong></td>
<td>• Plan for project-based learning</td>
</tr>
<tr>
<td>Responsible Decision-Making</td>
<td>• Assist students with discovering individual strengths</td>
</tr>
<tr>
<td></td>
<td>• Model and promote respecting differences</td>
</tr>
<tr>
<td></td>
<td>• Model and promote active listening</td>
</tr>
<tr>
<td></td>
<td>• Help students develop communication skills</td>
</tr>
<tr>
<td></td>
<td>• Demonstrate value for a diversity of opinions</td>
</tr>
</tbody>
</table>

After students work on a task solving real-world problems involving multiplication of fractions, ask the students to chart their solutions and hang them up around the room. Then have the students go around the room and write questions or positive feedback on sticky notes and place them on the charts.

Model and reinforce effective communication, relationship, and conflict-resolution skills that encourages students to feel comfortable taking mathematical risks.

Give students support and guidance as needed when they are working through a conflict. Ask nonjudgmental questions about what happened and what the students might do differently to avoid the conflict. This can be done effectively in teacher-led small group instruction during the Independent phase of the Balanced Math Instructional Model.
<table>
<thead>
<tr>
<th>Self-Awareness</th>
<th>Example practices that address Responsible Decision-Making:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-Management</td>
<td>• Support collaborative decision making for academics and behavior</td>
</tr>
<tr>
<td>Social-Awareness</td>
<td>• Foster student-centered discipline</td>
</tr>
<tr>
<td>Relationship Skills</td>
<td>• Assist students in step-by-step conflict resolution process</td>
</tr>
<tr>
<td>✓ Responsible Decision-Making</td>
<td>• Foster student independence</td>
</tr>
<tr>
<td></td>
<td>• Model fair and appropriate decision making</td>
</tr>
<tr>
<td></td>
<td>• Teach good citizenship</td>
</tr>
</tbody>
</table>

In mathematics classes, students can struggle with getting started on a challenging task. Teachers can motivate students’ learning of mathematics through opportunities for exploring problem solving that builds on and extends their current mathematical understanding.

Support students through the steps of making a decision anytime they face a choice or decision. (For example: Have the students refer to the anchor chart on strategies for multiplying a fraction by a whole number and decide which strategy they are most comfortable using.)
<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
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<tr>
<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
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<tr>
<td></td>
<td>Read directions aloud</td>
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<td>Color code materials</td>
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</tbody>
</table>

| Recall                                | Organization                                    |                                                   |                                             |
Differentiated Instruction

**Accommodate Based on Content Specific Needs:**

- Teacher modeling
- Anchor charts to model strategies
- Use technology to draw visual area fraction models.
- Use interactive technology and drawing programs to solve real-world fraction problems.
- Use a drawing program, word processing or spreadsheet to explain how a product is related to the magnitude of factors.
- Use a story context or visual fraction model to divide a unit fraction by a non-zero whole number.
- Use interactive technology to improve multiplication fact fluency and accuracy.
- Use manipulatives, a drawing program or interactive technology to explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of ten or represents a power of ten using whole number exponents.
- Use concrete models, drawings, and interactive technology to explain the reasoning used, relating the strategy to the written method.
- Chart academic vocabulary with visual representations.
- Students can utilize math journals to write notes, copy solution steps, and translate terms and key vocabulary if needed.
- Create and provide a list of properties of operations as strategies to add, subtract, multiply, and divide decimals.
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Science Connection:

Water Water: 5-ESS3-1
- Students compare the amount of water they use in daily life with the amount allotted for each person each day on a Space Shuttle. [https://science.nasa.gov/science-news/science-at-nasa/2000/ast02nov_1](https://science.nasa.gov/science-news/science-at-nasa/2000/ast02nov_1)

Social Studies Connection:

Road Trip: 6.1.8.B.5.a

English Language Arts Connection:

Writing a Division Story: W.5.3a, W.5.4
- Requiring students to write a division story stimulates imagination, supports language arts in math instruction and requires students to personalize the content into context. [http://sciencing.com/write-division-story-problem-8315698.html](http://sciencing.com/write-division-story-problem-8315698.html)
Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is…

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
## Assessments

### Required District/State Assessments
- Unit Assessments
- NJSLA
- SGO Assessments

### Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
<table>
<thead>
<tr>
<th>New Jersey Student Learning Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>5.NF.B.4</strong> Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.</td>
</tr>
<tr>
<td><strong>5.NF.B.4a</strong> Interpret the product ((a/b) \times q) as a parts of a partition of (q) into (b) equal parts; equivalently, as the result of a sequence of operations (a \times q \div b). For example, use a visual fraction model to show ((2/3) \times 4 = 8/3), and create a story context for this equation. Do the same with ((2/3) \times (4/5) = 8/15). (In general, ((a/b) \times (c/d) = ac/bd).)</td>
</tr>
<tr>
<td><strong>5.NF.B.4b</strong> Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.</td>
</tr>
<tr>
<td><strong>5.NF.B.5</strong> Interpret multiplication as scaling (resizing).</td>
</tr>
<tr>
<td><strong>5.NF.B.6</strong> Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.</td>
</tr>
<tr>
<td><strong>5.NF.B.7</strong> Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. <em>(benchmarked)</em></td>
</tr>
<tr>
<td><strong>5.NF.B.7a</strong> Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for ((1/3) \div 4), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that ((1/3) \div 4 = 1/12) because ((1/12) \times 4 = 1/3).</td>
</tr>
<tr>
<td><strong>5.NF.B.7b</strong> Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for (4 \div (1/5)), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (4 \div (1/5) = 20) because (20 \times (1/5) = 4).</td>
</tr>
<tr>
<td><strong>5.NF.B.7c</strong> Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?</td>
</tr>
<tr>
<td>New Jersey Student Learning Standards</td>
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<tr>
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<tr>
<td><strong>5.NBT.A.2</strong> Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</td>
</tr>
<tr>
<td><strong>5.NBT.B.7</strong> Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
</tr>
<tr>
<td><strong>5.MD.A.1</strong> Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</td>
</tr>
<tr>
<td>Mathematical Practices</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
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<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
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<tr>
<td>4. Model with mathematics.</td>
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<td>5. Use appropriate tools strategically.</td>
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<tr>
<td>6. Attend to precision.</td>
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<tr>
<td>7. Look for and make use of structure.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standards (NJSLS):
4.NF.B.4a, b, 5.NF.B.5, 5.NF.B.6, 5.NF.B.7*, 5.NBT.A.2*, 5.NBT.B.7*, 5.MD.A.1

Unit Focus:
- Apply and extend previous understandings of multiplication and division
- Understand the place value system
- Perform operations with multi-digit whole numbers and with decimals to hundredths
- Convert like measurement units within a given measurement system

New Jersey Student Learning Standards:
5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

5.NF.B.4b: Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Student Learning Objective 1: Multiply fractions by whole numbers and fractions by fractions, drawing visual models such as area models with the correct use of tiling, to represent products showing \((a/b) \times (c/d) = ab(1/bd)\), and creating story contexts.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>5.NF.4a-1</td>
<td>Students may believe that multiplication always results in a larger number and that division always results in a smaller number.</td>
<td>One factor in a multiplication problem tells you the number of groups and one tells you the number of items in the group, or one factor can tell you the number</td>
<td>IFL Task(s): “Multiplication with Fractions: Finding Portions of Numbers”</td>
</tr>
<tr>
<td>MP 2</td>
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<td>MP 3</td>
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<td>MP 4</td>
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<td>MP 5</td>
<td>MP 6</td>
<td>MP 7</td>
<td>MP 8</td>
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<tr>
<td>Tasks require finding a fractional part of a whole number quantity.</td>
<td>The result is equal to a whole number in 20% of tasks; these are practice forward for MP.7.</td>
<td>Tasks have “thin context” or no context.</td>
<td><strong>5.NF.4a-2</strong></td>
<td></td>
</tr>
<tr>
<td>The result is equal to a whole number in 20% of tasks; these are practice forward for MP.7.</td>
<td><strong>5.NF.4b-1</strong></td>
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</tbody>
</table>
| Tasks have “thin context” or no context. | 50% of the tasks present students with the rectangle dimensions and ask students to find the area; 50% of the tasks give the fractions and | Using models when multiplying with fractions will enable students to see that the results can be larger or smaller. | **Additional Tasks:**

**The Hiking Trail**

**Time for Recess**

**Basketball or Football**

**Folded Paper Lengths**

**Fundraiser Brownies**

**Model That Area**

**Examples:**

**Whole Number x Fraction model:**

![Whole Number x Fraction model](image)

**Fraction x Fraction model:**

![Fraction x Fraction model](image)

**Bar model:**

![Bar model](image)

of columns/groups and the items in the columns.

If the number of groups is less than one, the product will be smaller than the size of each group because the size of the group must be partitioned.

How can decomposing fractions or mixed numbers help us multiply fractions?

If a fraction is multiplied by a whole number or another fraction, will the product increase or decrease?

How are multiplying whole numbers and multiplying fractions the same and/or different?
| the product and ask students to show a rectangle to model the problem. | Use transparencies to model multiplication of fractions.

Represent children’s solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. \(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}, \ 3 \times \frac{1}{8} = \frac{3}{8} \text{ and } 3 \div 8 = \frac{3}{8}\).

Performing operations with fractions can be enhanced with the use of virtual manipulatives from websites such as NLVM.

Performing operations with fractions can be enhanced with the use of virtual manipulatives from websites such as NLVM.

Students examine the magnitude of products in terms of the relationship of two factors. They develop rules to determine the magnitude of products given the size of each factor.

Multiply fractional lengths to find areas of rectangles.

Represent fraction products as rectangular areas.

Compute the areas of a rectangle with fractional side lengths.

Tile a unit square into unit fraction side lengths. |
Prove through tiling the equivalence of multiplication and area.

**SPED Strategies:**
Review previous understandings of multiplication such as the importance and relationship of factors and what information they provide.

Break the information into steps or key components and monitor the student’s comprehension to assure that understanding is taking place as each step occurs.

Solve real-world problems using multiplication of fractions by whole numbers

**ELL Strategies:**
Explain how a product is related to the magnitude of the factors orally and in writing in L1 (student’s native language)
New Jersey Student Learning Standards:

5.NF.B.5: Interpret multiplication as scaling (resizing), by:

5.NF.B.5a: Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

5.NF.B.5b: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence \( \frac{a}{b} = \frac{(n \times a)}{(n \times b)} \) to the effect of multiplying \( \frac{a}{b} \) by 1.

Student Learning Objective 2: Explain how a product is related to the magnitude of the factors, including cases in which one factor is a fraction greater than 1 and cases in which one factor is a fraction less than 1.

Modified Student Learning Objectives/Standard: N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2 MP 4</td>
<td><strong>5.NF.5a</strong></td>
<td>Being able to estimate and mentally multiply whole numbers will allow students to determine reasonable answers</td>
<td>When multiplying with at least one factor greater than one, the product will be</td>
<td>IFLTasks:</td>
</tr>
<tr>
<td>MP 6</td>
<td>MP 7</td>
<td>completed without performing the indicated multiplication. ii) Products involve at least one factor that is a fraction or mixed number.</td>
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<tr>
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<td>and the relationship of the product to the factors used. Students examine the magnitude of products in terms of the relationship of two factors. This understanding can be used to develop rules that assist in determining the magnitude of products given the size of each factor.</td>
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<td>Students must investigate what happens to the product when one factor remains the same and one factor changes. Students can use concrete models (such as cuisenaire rods or circle pieces) and iconic models (such as shading rectangles) to connect these models to the equations.</td>
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<td>Compare the size of a product to the size of one of its factors, considering the size of the other factor (at least one factor is a fraction).</td>
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<td><strong>SPED Strategies:</strong> Teach from simple to complex, moving from concrete to representation to abstract at the student’s pace.</td>
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<td>Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).</td>
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<td>Have students work together to solve and then check their solutions.</td>
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<td>greater than at least one of the factors because more than one group of the other factor is being utilized.</td>
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<td></td>
<td>Multiplication and division can increase, decrease, or keep a number the same size.</td>
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<td>When multiplying two factors less than one, the product will be less than either factor because less than one whole group of less than one is being utilized (a portion of a portion).</td>
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<td></td>
<td>Why is it useful to compare numbers?</td>
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<td>How are fractions parts of a whole?</td>
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<td>“Multiplication with Fractions: Finding Portions of Numbers” <strong>PBAs:</strong> Sarah and Jose Multiplying Fractions on a Number Line <strong>Additional Tasks:</strong> Multiplication Three in a Row Preparing a Prescription Field Trip Funds Elmer’s Multiplication Error Featured Task Fundraising Multiplication and Scale Which room is larger</td>
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</tbody>
</table>

![Image](image_url)
**ELL Strategies:**

Vary the grouping in the classroom, such as sometimes using small group instruction to help ELLs learn to negotiate vocabulary with classmates and other times using native language support to allow a student to find full proficiency of the mathematics first.

Allow ELL students to talk to a peer in their native language when necessary to clarify understanding and clear up misunderstandings.
New Jersey Student Learning Standard:
5.NF.B.6: Solve real world problems involving multiplication of fractions and mixed numbers, e.g.by using visual fraction models or equations to represent the problem.

Student Learning Objective 3: Solve real-world problems involving multiplication of fractions (including mixed numbers), using visual fraction models or equations to represent the problem.

Modified Student Learning Objectives/Standard: N/A

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>5.NF.6-1</td>
<td>Represent children’s solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. (1/8 + 1/8 + 1/8 = 3/8, 3 x 1/8 = 3/8 and 3 divided by 8 is 3/8).</td>
<td>Multiplication does not always make the product larger than the factors and division does not always make the quotient smaller than the divisor. The order of the factors in a multiplication problem can be rearranged (flipped) because: the product is still the same. (Commutative Property). When multiplying with at least one factor greater than one, the product will be greater than at least one of the factors because more than one group of the other factor is being utilized.</td>
<td>IFL Tasks: “Multiplication with Fractions: Finding Portions of Numbers”</td>
</tr>
<tr>
<td>MP 2</td>
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<td>MP 8</td>
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</table>

5.NF.6-2
- Tasks present one or both factors in the form of a mixed number.

- Situations include area and comparison/times as much, with product unknown. (See Table 2, Common multiplication and division situations, p. 89 of CCSS.)
- Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.

- Allow students the opportunity to think about the reasonableness of their solutions in terms of the context and the numbers.

Additional Tasks:
- Measuring for a Pillow
- Multiplying Fractions with Color Tiles
- Comparing MPs
- Bake Sale
• Situations include area and comparison/times as much, with product unknown.
• Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.

Watch for misconceptions from previous multiplication and students who struggle understanding what information in a word problem indicates that they should multiply by providing more experiences with visual representations, breaking the problems into smaller parts, and explain their thinking as they complete each part of the problem.

Model multiplication of fractions through visual models.

Represent the product in simplest terms.

Solve word problems involving multiplication of fractions and mixed numbers.

Example:
There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many busses would it take to carry only the girls?

How can multiplying fractions be modeled using area, a number line, or measurement models?

Example:
There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many busses would it take to carry only the girls?

SPED Strategies:
Introduce essential terms and vocabulary prior to the mathematics instruction.
Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

Highlight critical vocabulary in discussion. For example, show a picture of ‘half.’

Connect language (fractions) with concrete and pictorial experiences (creating a number line, book marks with pictures representing fractions, fraction bars, fraction tower cubes, fraction circles. Sample fractions created out of paper plates, etc.)

**ELL Strategies:**
Teacher-talk with illustrative gestures.

Vary your voice to guide comprehension. Speak dynamically with expression. Make eye-to-eye contact and speak slowly and distinctly.

Vary the grouping in the classroom, such as sometimes using small group instruction to help ELLs learn to negotiate vocabulary with classmates and other times using native language support to allow a student to find full proficiency of the mathematics first.

View a video in native language about solving real-world problems.
New Jersey Student Learning Standard:
**5.NF.B.7a:** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \((1/3) ÷ 4\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \((1/3) ÷ 4 = 1/12\) because \((1/12) × 4 = 1/3\).

**Student Learning Objective 4:** Divide a unit fraction by a non-zero whole number and interpret by creating a story context or visual fraction model.

**Modified Student Learning Objectives/Standard:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>5.NF.7a</td>
<td>This is the students’ first experience with division of fractions. They use their understanding of whole number division to visualize what happens when they are dividing whole numbers by fractions and fractions by whole numbers. Review partitive model of division of whole numbers. Give students problems to solve and have them explain their thinking by relating the division of a fraction by a whole number. Look for connections between divisions with fractions using previous experiences with the relationship between multiplication and division. Different real world interpretations can be associated with division calculations involving fractions. Facilitate discussions that illuminate different</td>
<td>To divide a number, (a), by another number, (b), means to determine how many iterations of (b) fit inside (a), including when (a) and (b) are fractions or decimals. A fraction describes the division of a whole (region, set, segment) into equal parts. What does dividing a unit fraction by a whole number look like? Define a unit fraction as fraction with a numerator of 1.</td>
<td>IFL Tasks: “Dividing Fractions: Understanding Division with Numbers Less than 1”</td>
</tr>
<tr>
<td>MP 2</td>
<td>5.C.2-4</td>
<td>Base explanations/reasoning on the relationship between multiplication and division. Content Scope: Knowledge and skills articulated in 5.NF.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP 3</td>
<td>5.C.5-3</td>
<td>Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 5.NF.7a, 5.NF.7b</td>
<td></td>
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<tr>
<td>MP 4</td>
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<td>MP 5</td>
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<td>MP 7</td>
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<tr>
<td>MP 8</td>
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</tbody>
</table>

**IFL Tasks:**
“Dividing Fractions: Understanding Division with Numbers Less than 1”

**PBAs:**
Charlie’s Number Line Cups of Flour

**Additional Tasks:**
Divide a Unit by a Fraction Get Off the Bus Creating Stories Writing a Division Story Improving our Schools
viewpoints students have that explain dividing fractions.

Have students compare and discuss two problems (one that divides a fraction by a whole and one that divides a whole by a fraction). Students should express their understanding in writing, with a pair and in a whole group setting to reveal multiple viewpoints.

Provide students with representations and ask them to decide if the representations are accurate.

Emphasize the use of visual models to understand the dividing of fractions that is taking place.

One misconception to be addressed is that students always have to “divide the bigger number by the smaller number”. Make the connection to the structure of a fraction in which \( \frac{3}{4} \) also means 3 divided by 4.

Draw/show division of a unit fraction by a whole number as dividing the unit fraction into smaller parts.

The inverse relationship between multiplication and division can be used to divide with fractions.
For Example: $\frac{1}{3} \div 4$

![Fraction Division Diagram]

Explain the effects of dividing a unit fraction by a whole number.

**SPED Strategies:**
Create a visual model of the lesson expectations as a class.

Review prerequisite skills before introducing new concepts (multiplication and division facts, identifying fractions, dividing whole numbers and multiplying whole numbers by a fraction)

Utilize interactive journals to review examples of previous concepts.

**ELL Strategies:**
Review in L1 (student’s native language) content-specific vocabulary words (i.e., quotient, equivalent, divisor).
New Jersey Student Learning Standard:
5.NF.B.7b:* Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div \frac{1}{5}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{5} = 20$ because $20 \times \left(\frac{1}{5}\right) = 4$.

Student Learning Objective 5: Divide a whole number by a unit fraction and interpret by creating a story context or visual fraction model.

Modified Student Learning Objectives/Standard: N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>5.NF.B.7</td>
<td>Using the measurement meaning of division (looking for the number of groups that can be made when the total and the number of items in a group are shown) will help students to visualize and model what is happening when dividing a whole number by a fraction.</td>
<td>To divide a number, $a$, by another number, $b$, means to determine how many iterations of $b$ fit inside $a$, including when $a$ and $b$ are fractions or decimals.</td>
<td>IFL Tasks: “Dividing Fractions: Understanding Division with Numbers Less than 1”</td>
</tr>
<tr>
<td>MP 2</td>
<td>5.C.2-4</td>
<td>Define the reciprocal of a unit fraction for the purpose of division.</td>
<td>A fraction describes the division of a whole (region, set, segment) into equal parts.</td>
<td>Additional Tasks: Origami Stars</td>
</tr>
<tr>
<td>MP 3</td>
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<td>Simplify quotients to the lowest term.</td>
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<td>MP 4</td>
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</table>

Create a visual model as a class to use as a future reference.

Utilize interactive journals to review examples of previous concepts and to demonstrate growth in math writing and reasoning.

Teach in small chunks so students get a lot of practice with one step at a time.
<table>
<thead>
<tr>
<th>5.C.5-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 5.NF.7a, 5.NF.7b</td>
</tr>
</tbody>
</table>

Discuss patterns students see in written equations and connect those equations to the relationship between multiplication and division.

\[ 3 \div \frac{1}{4} = \_ \_ \_ \_ \quad \frac{1}{4} \times \_ \_ \_ \_ \_ = 3 \]

Model questions that students can ask themselves to help interpret the problem. For example, how many groups of \( \frac{1}{2} \) can I make out of 8? This question can be related to the visual representation students use to solve the problem.

The use of models and drawings will eliminate the confusion that eight divided by on-half (\( 8 \div \frac{1}{2} \)) and eight divided by half (\( 8 \div 2 \)) produce the same results.

Have students compare and discuss two problems (one that divides a fraction by a whole and one that divides a whole by fraction).

The inverse relationship between multiplication and division can be used to divide with fractions.

| Divide a Whole Number by a Unit Fraction |
Students should express their understanding in writing, with a pair and in a whole group setting to reveal multiple viewpoints.

Students will require experiences with fractional numbers that result in whole-number products and quotients.

Define the reciprocal of a unit fraction for the purpose of division.

Students can use a number line to develop an understanding of dividing a whole number by a unit fraction. For example:

SPED Strategies:
Review prerequisite skills before introducing new concepts.
Utilize interactive journals to review examples of previous concepts.
Teach in small chunks so students get a lot of practice with one step at a time.

ELL Strategies:
Re-teach the same concept with a variety of fluency games.
Allow students to lead group and pair-share activities. Utilize interactive journals to review examples of previous concepts and to demonstrate growth in math writing and reasoning.

**New Jersey Student Learning Standard:**
5.NF.B.7c: Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

**Student Learning Objective 6:** Solve real-world problems involving division of unit fractions by whole numbers or whole numbers by unit fractions.

**Modified Student Learning Objectives/Standard:** N/A

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>5.NF.7c</td>
<td>Using key words is not helpful and removes making sense from the process. Rather have students model the problem using pictures and ask supporting questions, such as “What do you know? What do you want to find out? How can you show that in your picture?” The real world situation determines how a remainder needs to be interpreted when solving a problem.</td>
<td>To divide a number, $a$, by another number, $b$, means to determine how many iterations of $b$ fit inside $a$, including when $a$ and $b$ are fractions or decimals. Dividing a number by a whole number results in a quotient that is a smaller number than the dividend because the dividend is being partitioned into parts.</td>
<td>IFL Tasks: “Dividing Fractions: Understanding Division with Numbers Less than 1” Additional Tasks: Salad Dressing Standing in Line Fabric Shop</td>
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<tr>
<td>MP 2</td>
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<td>MP 8</td>
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<tr>
<td>5.C.2-4</td>
<td>Explain the effects of dividing a whole number by a unit fraction (vice versa) in the context of a word problem.</td>
<td>Explain the effects of dividing a whole number by a unit fraction (vice versa) in the context of a word problem. Students may struggle determining which number goes where in the division problem. “Am I dividing the fraction by the whole number or the whole number by the fraction?” Drawing a picture using the information in the problem and focusing on what they want to find out will help. Simplify quotients to the lowest forms. Justify the reasonableness of answers in terms of the context of the problem. <strong>SPED Strategies:</strong> Color-code or highlight key words in math word problems. Let the student use a flowchart to plan strategies for problem solving. Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models (e.g., ten-frame); <strong>ELL Strategies:</strong> Have students paraphrase and write complex concepts in their own words (individually, pairs, or whole class) in their interactive math journal to</td>
<td>When dividing a number less than one, the quotient will be more than the dividend because either: you are making groups of an amount less than 1; or you are making less than one group. A fraction describes the division of a whole (region, set, segment) into equal parts. How does my knowledge of division of whole numbers apply to the division of fractions? What do I need to know and be able to do to divide fractions by whole numbers? <strong>Betty Bloom’s Party</strong> How does the Garden Grow</td>
<td></td>
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</table>

Betty Bloom’s Party How does the Garden Grow
**New Jersey Student Learning Standard:**

5.NBT.A.2: * Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

**Student Learning Objective 7:** Explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10; represent powers of 10 using whole-number exponents.

**Modified Student Learning Objectives/Standard:**

M.EE.5.NBT.A.2: Use the number of zeros in numbers that are powers of 10 to determine which values are equal, greater than, or less than.

<table>
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<tr>
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<tbody>
<tr>
<td>MP 2</td>
<td>5.NBT.2-2</td>
<td>Students reason that not just that the decimal point is moving but that you are multiplying or making the number 10 times greater a number of times. Since we are multiplying by a power of 10, the decimal point moves to the right. A pattern is created when a number is multiplied by a power of 10 and show</td>
<td>Multiplying a whole number by power of 10 will result in a product with as many 0s at the end as were in the power of 10. Sets of ten, one hundred, and so forth must be perceived as single entities when interpreting numbers using place value (e.g.,</td>
<td>Distance from the Sun</td>
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<tr>
<td>MP 6</td>
<td></td>
<td></td>
<td></td>
<td>What comes next?</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td></td>
<td></td>
<td>Multiplying a Whole Number by a Power of 10</td>
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</tbody>
</table>
about the place value system, in ways consistent with the indicated content scope.

| their solutions through multiple representations. Sets of ten, one hundred and so forth must be perceived as single entities when interpreting numbers using place value (e.g., 1 hundred is one group, it is 10 tens or 100 ones).

| Write whole number exponents to denote powers of 10. Illustrate and explain a pattern for how multiplying or dividing any decimal by a power of 10 relates to the placement of the decimal point. Connections to literacy can also be made with books about large numbers, such as: *How Much is a Million* by Steven Kellogg, *A Million Dots* by Andrew Clements, or *Big Numbers and Pictures That Show Just How Big They Are* by Edward Packard and Sal Murdocca.

| SPED Strategies: Provide students with a choice board that requires students to solve problems from simple to complex. Encourage students to explain their reasoning both orally and in writing. Draw a powers of 10 chart as a class and display and provide examples to clear up

| 1 hundred is one group, it is 10 tens or 100 ones). When multiplying a number times a power of ten, the exponent does not indicate the number of zeroes in the product should be emphasized. What patterns occur in our number system? What happens when we multiply a number by powers of ten?
mathematical misconceptions and generate conversation around the work. Highlight those examples such as 10 pointing out its equality to $10 \times 10 \times 10 \times 10 \times 10$ but not to $10 \times 5$ or even 5.

Allowing students to explore with a calculator and highlighting the functions used to calculate these expressions (e.g., 10 versus $10 \times 5$) can be valuable.

**ELL Strategies:**
Create and utilize an interactive math journal with pictures, specific vocabulary words, and to review examples of previous concepts and to demonstrate growth in math writing and reasoning.

Using L1 (student’s native language), review academic vocabulary to build background knowledge.

Provide students with a choice board that requires students to solve problems from simple to complex.
New Jersey Student Learning Standard:
5.NBT.B.7:* Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Student Learning Objective 8: Add, subtract, multiply, and divide decimals to hundredths using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; explain the reasoning used, relating the strategy to the written method.

Modified Student Learning Objectives/Standard:
M.EE.5.NBT.6-7 Illustrate the concept of division using fair and equal shares.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2 MP 3 MP 4 MP 5 MP 7</td>
<td>5.NBT.7-1 • Tasks do not have a context. • Only the sum is required. Explanations are not assessed here. • Prompts may include visual models, but prompts must also present the addends as numbers, and the answer sought is a number, not a picture. • Each addend is greater than or equal to 0.01 and less than or equal to 99.99.</td>
<td>Draw diagrams or use number reasoning for each situation. Model each word problem with diagrams and/or numbers. Use knowledge of repeated addition of decimal composites along with decomposition of decimal composites to solve problems. Use manipulatives, drawings, or equations to represent how many of a certain decimal unit comprise one whole.</td>
<td>If there is less than a whole number of groups, (e.g., 0.5 groups or 0.25 groups), the product will be less than the amount in the group and the number of groups because less than one whole group is being utilized. If there is more than a whole number of groups, the product will be greater than the amount in the group because more than one group of the decimal fraction/composite unit is being utilized.</td>
<td>John’s Race Family Meals Betty Bloom’s Party Creating Stories Divide a Unit Fraction by a Whole # Fabric Shop Get off the Bus</td>
</tr>
</tbody>
</table>
- 20% of cases involve a whole number—either the sum is a whole number, or else one of the addends is a whole number presented without a decimal point. (The addends cannot both be whole numbers.)

**5.NBT.7-2**
- Tasks do not have a context.
- Only the difference is required.
- Prompts may include visual models, but prompts must also present the subtrahend and minuend as numbers, and the answer sought is a number, not a picture.
- The subtrahend and minuend are each greater than or equal to 0.01 and less than or equal to 99.99. Positive differences only. (Every included subtraction problem is an unknown-addend problem included in 5.NBT.7-1.)
- 20% of cases involve a whole number—either the

**Example:**

4 – 0.3

The wholes must be divided into tenths.

<table>
<thead>
<tr>
<th>0.22 x 5</th>
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<tr>
<th>2.4 ÷ 4</th>
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<table>
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<tr>
<th>1.6 ÷ 0.2</th>
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</table>

Estimate decimal computation before computing with pencil and paper.

When answering a division problem involving a whole quotient, it is important for students to be able to decide whether

Either or both factors can be decomposed to form equivalent values (e.g., 2.3 x 6 is the same as 2 x 6) + (0.3 x 6).

Any representation (e.g., area model/arrays, number lines, set, or equations) of repeated addition/multiplication of a number by a number illustrates the number of groups, the size of each group (i.e., the unit or non-unit decimal fraction), and the resulting product or partial products.

When dividing by a decimal number less than one, the quotient will be more than the dividend because either; you are making groups of an amount less than one; or you are making less than one group.

Any representation of the division of a number divided by a number (area model, number line, or set model) highlights the starting amount, the final
<table>
<thead>
<tr>
<th>difference is a whole number, or the subtrahend is a whole number presented without a decimal point, or the minuend is a whole number presented without a decimal point. (The subtrahend and minuend cannot both be whole numbers.)</th>
<th>the context requires the result to be reported as a whole number with remainder or a mixed number/decimal.</th>
<th>amount, and the impact of the division.</th>
</tr>
</thead>
<tbody>
<tr>
<td>tasks do not have a context.</td>
<td><strong>SPED Strategies:</strong> Reduce length of assignment and different instructional mode of delivery Increase one-on-one time. Utilize working contact between you and student at risk.</td>
<td>What are some ways you can add, subtract, multiply and divide decimals?</td>
</tr>
<tr>
<td>only the product is required.</td>
<td><strong>ELL Strategies:</strong> Model the process. Talk aloud while solving problems on the overhead or chalkboard to show the thinking process and common errors. Have students explain their thinking process aloud to a classmate while solving a problem. Integrate reading and writing through the use of journals, learning logs, poems, literature, etc.</td>
<td></td>
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</tbody>
</table>
5.NBT.3, and see p. 17 of the Number and Operations in Base Ten Progression document.

- Problems are 2-digit x 2-digit or 1-digit by 3- or 4-digit. (For example, 7.8 x 5.3 or 0.3 x 18.24.)
- 20% of cases involve a whole number—either the product is a whole number, or else one factor is a whole number presented without a decimal point. (Both factors cannot both be whole numbers.)

**5.NBT.7-4**

- Tasks do not have a context.
- Only the quotient is required.
- Prompts may include visual models, but prompts must also present the dividend and divisor as numbers, and the answer sought is a number, not a picture.
- Divisors are of the form XY, X0, X, X.Y, 0.XY, 0.X, or 0.0X (cf. 5.NBT.6), where X and Y
represent non-zero digits. Dividends are of the form
XY, X0, X, XYZ.W,
XY0.Z, X00.Y, XY.Z,
X0.Y, X.YZ, X.Y, X.0Y,
0.XY, or 0.0X, where X,
Y, Z, and W represent
non-zero digits.

- Quotients are either
  whole numbers or else
decimals terminating at
the tenths or hundredths
place. (Every included
division problem is an
unknown-factor problem
included in 5.NBT.7-3.)
**New Jersey Student Learning Standard:**

5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

**Student Learning Objective 9:** Convert standard measurement units within the same system (e.g., centimeters to meters) in order to solve multi-step problems.

**Modified Student Learning Objectives/Standard:**

M.EE.MD.1.a: Tell time using an analog or digital clock to the half or quarter hour.
M.EE.MD.1.b: Use standard units to measure weight and length of objects.
M.EE.MD.1.c: Indicate relative value of collections of coins.

<table>
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<tbody>
<tr>
<td>MP 1</td>
<td>5.MD.1-1</td>
<td>Incorporate teaching situations that allow students to realize that they need conversion information rather than giving it to them upfront; having students identify what information they need to have to solve the problem and knowing where to go to find it allows them to engage in the Standard for Mathematical Practice 5: Use appropriate tools strategically.</td>
<td>Length measurements will require regrouping, such as feet and inches that can be added or subtracted where 1 foot is regrouped as 12 inches.</td>
<td>Estimate, Measure, Estimate</td>
</tr>
<tr>
<td>MP 2</td>
<td>5.MD.1-2</td>
<td>Students will need to be familiar with benchmarks and conversion factors for units of liquid volume, weight, and mass. Engage students in as many activities in which they measure something with a specified unit.</td>
<td>The longer the unit of measure, the fewer units it takes to measure an object.</td>
<td>Water Water</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td></td>
<td>The magnitude of the attribute to be measured and the accuracy needed determines the measurement unit.</td>
<td>Minutes and Days</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td></td>
<td>Relationships between measurement units of the same length can be expressed as an equation (e.g., 1 ft = 12 in; 1 m = 100 cm). Relationships exist that enable you to convert between units of length by multiplying or dividing.</td>
<td>Converting Fractions</td>
</tr>
<tr>
<td></td>
<td>Multi-step problems must have at least 3 steps.</td>
<td></td>
<td></td>
<td>Fruits and Vegetables</td>
</tr>
</tbody>
</table>
and then measure it again with a different related unit.

Focus on helping students convert measurements into larger or smaller units within a measurement system by reinforcing place value for whole numbers and decimals and then focus on the connection between fractions and decimals.

Begin problem solving with simple problems that focus on renaming units to represent the solution before experiencing problems that require renaming to find the solution.

Model vocabulary usage with measurement terms, including conversion/convert, metric and customary measurement, and from previous grades, relative size, liquid volume, mass, and length.

**SPED Strategies:**
Let students choose their mode of response: written, oral, concrete, pictorial, or abstract.

Provide students with the opportunity to use rulers and other types of tools to measure objects in the classroom.

Relationships between measurement units of the same capacity can be expressed as a ration (e.g., 1 qt to 2 pt or 1 qt = 1 pt; 1 L to 1,000 mL or 1 L = 1,000 mL).

Relationships between measurement units of weight/mass can be expressed as a ration (e.g., 1 lb to 16 oz or 1 lb = 16 oz; 1 kg to 1,000 g or 1 kg = 1,000 g).

How do we convert measurements within systems?
**ELL Strategies:**
Allow ELL students to talk to a peer in their native language when necessary to clarify understanding and clear up misunderstandings.

Utilize interactive journals to review examples of previous concepts and to demonstrate growth in math writing and reasoning.
### Integrated Evidence Statements

#### 5.NBT.A.Int: Demonstrate understanding of the place value system by combining or synthesizing knowledge and skills articulated in 5.NBT.Int.1:

- Perform exact or approximate multiplications and/or divisions that are best done mentally by applying concepts of place value, rather than by applying multi-digit algorithms or written strategies.
  - Tasks do not have a context.

#### 5.NBT.Int.1: Solve one-step word problems involving multiplying multi-digit whole numbers.

- The given factors are such as to require an efficient/standard algorithm (e.g., \(726 \times 4871\)). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as \(7250 \times 400\)).
- For purposes of assessment, the possibilities for multiplication are 1-digit x 2-digit, 1-digit x 3-digit, 2-digit x 3-digit, 2-digit x 4-digit, or 3-digit x 3-digit.
- Word problems shall include a variety of grade-level appropriate applications and contexts.

#### 5.Int 2: Solve word problems involving multiplication of three two-digit numbers.

- The given factors are such as to require an efficient/standard algorithm (e.g., \(76 \times 48 \times 39\)). Factors in the task do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as \(50 \times 20 \times 15\)).
- Word problems shall include a variety of grade-level appropriate applications and contexts.

#### 5.C.5-3: Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response).

#### 5.C.7-4: Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.)

- Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 5.

#### 5.D.2: Solve multi-step contextual problems with degree of difficulty appropriate to Grade 5, requiring application of knowledge and skills articulated in 4.OA, 4.NBT, 4.NF, 4.MD

- Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 5.
- Multi-step problems must have at least 3 steps.
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Fractional Sides</th>
<th>Minute</th>
<th>Second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argument</td>
<td>Hour</td>
<td>Mixed Numbers</td>
<td>Side Lengths Square Units</td>
</tr>
<tr>
<td>Centimeter (cm)</td>
<td>Gallon (gal)</td>
<td>Models</td>
<td>Representations</td>
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<tr>
<td>Columns</td>
<td>Gram (g)</td>
<td>Numerator</td>
<td>Standard Measurement</td>
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<tr>
<td>Compose</td>
<td>Hundredths</td>
<td>Ounce (oz)</td>
<td>Tens</td>
</tr>
<tr>
<td>Contexts</td>
<td>Inch (in)</td>
<td>Partition</td>
<td>Unit Fraction</td>
</tr>
<tr>
<td>Cup (c)</td>
<td>Kilogram (kg)</td>
<td>Pint (pt)</td>
<td>Visual Fraction Models</td>
</tr>
<tr>
<td>Decimeters</td>
<td>Kilometer (km)</td>
<td>Place Value</td>
<td>Volume</td>
</tr>
<tr>
<td>Decompose</td>
<td>Length</td>
<td>Pound (lb)</td>
<td>Yard (yd)</td>
</tr>
<tr>
<td>Denominator</td>
<td>Liquid</td>
<td>Powers</td>
<td></td>
</tr>
<tr>
<td>Estimate</td>
<td>Liter (L)</td>
<td>Properties of Operations</td>
<td></td>
</tr>
<tr>
<td>Equal Parts</td>
<td>Manipulatives</td>
<td>Quot (qt)</td>
<td></td>
</tr>
<tr>
<td>Exponents</td>
<td>Mass</td>
<td>Reasonable</td>
<td></td>
</tr>
<tr>
<td>Foot (ft)</td>
<td>Meter (m)</td>
<td>Reasoning</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Mile (mi)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction</td>
<td>Milliliter (mL)</td>
<td></td>
<td></td>
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## References & Suggested Instructional Websites

<table>
<thead>
<tr>
<th>Website</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>National Council of Teachers of Mathematics</td>
<td>This website contains activities and lessons, and virtual manipulatives organized by strand. <a href="http://illuminations.nctm.org">http://illuminations.nctm.org</a></td>
</tr>
<tr>
<td>Internet For Classrooms</td>
<td>This site is a list of math sites for lessons and teacher tools. <a href="http://illuminations.nctm.org">www.internet4classrooms.com</a></td>
</tr>
<tr>
<td>The Georgia Standards</td>
<td>contain exceptional tasks and curriculum support. <a href="http://illuminations.nctm.org">www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx</a></td>
</tr>
<tr>
<td>Illustrative Mathematics</td>
<td>is a library of tasks linked to Common Core State Standards. <a href="http://illuminations.nctm.org">www.illustrativemathematics.org/</a></td>
</tr>
<tr>
<td>Sumdog</td>
<td>is a website that contains free practice and contests throughout the year. <a href="http://www.sumdog.com">http://www.sumdog.com</a></td>
</tr>
<tr>
<td>K-5 Math Teaching Resources</td>
<td>site contains free math teaching resources, games, activities and journal tasks. <a href="http://www.k-5mathteachingresources.com">http://www.k-5mathteachingresources.com</a></td>
</tr>
<tr>
<td>MobyMax</td>
<td>program is noted for its easy to use format for remedial and challenging assignments. <a href="http://www.mobymax.com">http://www.mobymax.com</a></td>
</tr>
<tr>
<td>Study Island</td>
<td>Study Island supports the learning process and builds off of your students’ enthusiasm for technology with engaging, interactive lessons and activities. Students can work through the web-based program at their own pace, or teachers can guide students through the program. <a href="http://www.studyisland.com/">http://www.studyisland.com/</a></td>
</tr>
</tbody>
</table>
Field Trip Ideas

MoMath/ Museum of Mathematics:  Mathematics illuminates the patterns and structures all around us. Our dynamic exhibits gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger), but with a special emphasis on activities for 4th through 8th graders. 
http://momath.org/

Liberty Science Center:  Student mastery of STEM (Science, Technology, Engineering, and Mathematics) has never been more important. Under the newest national standards, educators are required to instruct students in science and technology with active question-and-answer pedagogy and hands-on investigation. Liberty Science Center understands educators’ needs and offers a full portfolio of age-appropriate, curriculum-linked STEM programs suitable for preschoolers through technical school students, including pupils with special needs. 
http://lsc.org/for-educators/

Discovery Times Square: New York City’s first large-scale exhibition center presenting visitors with limited-run, educational and immersive exhibit experiences while exploring the world’s defining cultures, art, history, and events. 
http://discoverymuseum.org/

Great Falls National Park:  A Revolutionary Idea: Cotton & silk for clothing; locomotives for travel; paper for books & writing letters; airplanes, & more. What do they have in common? They all came from the same place - Paterson, NJ. 
https://www.nps.gov/pagr/index.htm

Passaic County Historical Society Lambert Castle Museum: The museum consists of mostly self-guided exhibits. Tours of the first floor will be offered every half hour (as interest permits) or as visitors arrive on weekdays. If you are interested in a tour of the first floor, please let our docent know when you arrive. 
http://www.lambertcastle.org/museum.html

The Paterson Museum: Paterson Museum is a museum in Paterson, in Passaic County, New Jersey, in the United States. Founded in 1925, it is owned and run by the city of Paterson and its mission is to preserve and display the industrial history of Paterson.